SPIN EFFECTS IN TWO QUARK SYSTEM AND MIXED STATES

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Abstract

Based on the numeric solution of a system of coupled channels for vector mesons (S- and D-waves mixing) and for tensor mesons (P- and F-waves mixing) mass spectrum and wave functions of a family of vector mesons $q\overline{q}$ in triplet states are obtained. The calculations are performed using a well known Cornell potential with a mixed Lorentz-structure of the confinement term. The spin-dependent part of the potential is taken from the Breit-Fermi approach. The effect of singular terms of potential is considered in the framework of the perturbation theory and by a configuration interaction approach (CIA), modified for a system of coupled equations. It is shown that even a small contribution of the D-wave to be very important at the calculation of certain characteristics of the meson states.

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1 Introduction

Meson states, which are considered as the bound states of a quark-antiquark system, are convenient objects for studying both the strong interaction effects and various characteristics of weak interaction [1,2]. For the description of the low-energy properties of hadrons the following approaches are used: Bete-Solpiter approach, lattice calculation technique and potential models. Each of these methods has its own advantages and shortcomings. The potential models are the simplest from the point of view of mathematics, which is essential for practical calculations. In the framework of the present model the averaged mass spectrum [1], spin effects [3,4] and the decay widths of heavy quarkonia [5] can be well described. As concerning the light-quark systems, the situation is rather contradictory. Using the same parameters, some of the effects (spin effects, decay, averaged mass spectrum) can be described [6], but not all the effects together. The reason for this is the fact the light-quark systems being explicitly relativistic, and relativistic potential models should be applied for them.

When the spin effects are considered in potential models (in Breit-Fermi approach), singular terms of the interquark interaction potential of the form $\frac{1}{r^3}$ and $\delta(\mathbf{r})$ arise. This is a serious problem at the calculations of the meson characteristics. As a rule, in such case perturbation theory is used [7]. But it has certain shortcomings. The main disadvantage is that the theory supposes small interaction, but in hadron physics it is the perturbation of the order 30-500 MeV, what is comparable with the distance between the unperturbed energy levels, hence, the condition of small perturbation not being fulfilled.The variational method also very popular in hadron physics [8]. These methods were in fact the first, which used to investigate the characteristics of mesons, considered as two-quark systems.

In the triplet state configurations with uncertain orbital moment $L = J+1$, $L = J-1$ occur. For example, mixing of S- and D-waves occurs in the state 1^- and P- and F- mixing for the state 2^+ (we use the spectroscopic notations J^P where the total moment and the system parity are indicated). In most papers where the triplet states are considered (See Ref. 3 and references therein) the authors neglect the channel coupling or introduce an additional parameter - the mixing angle [9-11].

In papers [12] the mixed state 1^- for $q\overline{q}$-systems was considered in the frame of coupled channel and mass spectra with lepton decay width was investigated. It was observed a small D-wave mixing, but this small contribution causes essential influence on the value of hyperfine splitting and decay width.

In the present paper the effect of the singular terms of the interquark interaction potential for the quarkonium state 1^-, described by a system of coupled equations, is studied and the comparison of perturbation methods and configuration interaction, modified for the coupled equation system, is performed. We followed Ref. [12], when choose the Lorentz structure and parameters of $q\overline{q}$-interaction with the numerical solution scheme of the coupled differential equation. The calculation version of the configuration interaction approach based on the Schroedinger equation is taken from Ref. 3, where the hyperfine splitting is considered. In such approach the mixing angle for S- and D-waves is determined by the system dynamics and does not require any additional experimental data.

2 Breit-Fermi approach

It is widely accepted that the interaction between two quarks (or heavy quark-antiquark) consist of a short range part describing the one-gluon exchange and a infinitely rising long-range part responsible for confinement of the quarks

$$V_0(r) = -\frac{\alpha_S}{r} + \beta r. \tag{1}$$

Wilson loop techniques suggest that the confining potential should be purely scalar, but relativistic potential calculations which have been published [13-15] show a need for (some) vector confinement. Maintaining $V_V(r) + V_S(r)$ unchanged we do allow a fraction of vector confinement [6,13,14]. Namely

$$V_V(r) = -\frac{\alpha_S}{r} + \beta_V r, \quad V_S(r) = \beta_S r \quad (\beta_V + \beta_S = \beta) \tag{2}$$

The confining potential transforms as the Lorentz scalar and vector potential transforms as the time component of a four-vector potential. As we can see, the choice of Lorentz structure of potential for quark-antiquark interaction is important model for study of spin effects [7,14-16].

We consider the Breit-Fermi Hamiltonian for case $m_1 = m_2 = m$

$$H = \frac{p^2}{m} + V_0 + H_{LS} + H_{SS} + H_T \tag{3}$$

with spin-orbit term

$$H_{LS} = \frac{1}{2mr^2}\left(3\frac{dV_V}{dr} - \frac{dV_S}{dr}\right)(\mathbf{LS}) \tag{4}$$

where $\mathbf{S} = \mathbf{S_1} + \mathbf{S_2}$ is the total spin of bound state and $\mathbf{L}$ is the relative angular momentum of its constituents, the spin-spin term

$$H_{SS} = \frac{2}{3m^2}\Delta V_V(r)\mathbf{S_1 S_2} \tag{5}$$

and the tensor term

$$H_T = \frac{1}{12m^2}\left(\frac{1}{r}\frac{dV_V}{dr} - \frac{d^2V_S}{dr^2}\right) S_{12} \tag{6}$$

where

$$S_{12} = 12\left(\frac{(\mathbf{S_1 r})(\mathbf{S_2 r})}{r^2} - \frac{1}{3}\mathbf{S_1 S_2}\right) \tag{7}$$

For bound state constituents of spin $\mathbf{S_1} = \mathbf{S_2} = \frac{1}{2}$, the scalar product of their spin, $\mathbf{S_1 S_2}$, is given by $\mathbf{S_1 S_2} = -\frac{3}{4}$ for singlet states ($S = 0$), and $\mathbf{S_1 S_2} = \frac{1}{4}$ for triplet states ($S = 1$).

Taking into account (2), then (4), (5) and (6) yield

$$V_{LS} = \frac{1}{2m_q m_{\overline{q}}}\left(3\frac{\alpha_S}{r^3} + 3\frac{\beta_V}{r} - \frac{\beta_S}{r}\right)(\mathbf{LS})$$

$$V_{SS} = \frac{4}{3m_q m_{\overline{q}}}\left(\frac{\beta_V}{r} - 2\pi\alpha_S\delta(r)\right)(\mathbf{S_1 S_2})$$

$$V_T = \frac{1}{m_q m_{\overline{q}}}\left(\frac{\alpha_S}{r^3} + \frac{\beta_V}{r}\right) S_{12}$$

The physical states of $q\bar{q}$ system are determined by total momentum J and its projection M_J, parity P and total spin S (Table 1), here we use spectroscopic notation $^{2S+1}L_J$

Table 1.States of two fermion system

	Singlet states $(S=0)$		Triplet states $(S=1)$	
J/P	+	-	+	-
0	- - - -	$^1S_0$	$^3P_0$	- - - -
1	$^1P_1$	- - - -	$^3P_1$	$^3S_1 +^3 D_1$
2	- - - -	$^1D_2$	$^3P_2 +^3 F_2$	$^3D_2$

It is seen from Table 1 that there are singlet and triplet states with definite orbital moment and triplet states with mixed orbital components ($L = J \pm 1$). For pure states wave functions are of the form $(F(r)/r)\,\mathcal{Y}^M_{JLS}$ where $\mathcal{Y}^M_{JLS}$ is the spin-orbital part of the wave function.

The radial part of wave function $F(r)$ for singlet states satisfies equation

$$\frac{d^2F}{dr^2} + [k^2 - \frac{L(L+1)}{r^2} -^1 \upsilon_c]F = 0 \tag{8}$$

where $^1\upsilon_c = m^1V_0$; $\;^1V_0 = V_V(r) + V_S(r) - (3/4)V_{SS}$ and $k^2 = mE$.

The radial function of the pure triplet states $^3P_0$ and states with $L = J$ obeys the same equation that singlet states (8), but with potentials $^3V_0 = V_V + V_S + \frac{1}{4}V_{SS} - 2V_{LS} - 4V_T$ and $^3V_0 = V_V + V_S + \frac{1}{4}V_{SS} - V_{LS} + 2V_T$ respectively.

The wave function for ground triplet state of $q\bar{q}$ system with negative parity ($P = -1$) is a mixture of states $^3S_1$ and $^3D_1$ and may be put in the form

$$\psi = \psi_S + \psi_D \equiv \frac{1}{r}u(r)\mathcal{Y}^1_{101} + \frac{1}{r}w(r)\mathcal{Y}^1_{121} \tag{9}$$

Then the equation $(H - E)\psi = 0$ is equivalent to coupled system [17]

$$[-\frac{1}{m}\frac{d^2}{dr^2} - E +^3 V_c]u + \sqrt{8}V_T w = 0,$$

$$[-\frac{1}{m}\frac{d^2}{dr^2} - E + \frac{6}{mr^2} +^3 V_c - 2V_T - 3V_{LS}]w + \sqrt{8}V_T u = 0, \tag{10}$$

where

$$^3V_c = V_V + V_S + \frac{1}{4}V_{SS}.$$

The Schroedinger equations are linked due to the presence of a tensor component V_T in the interaction potential. In Ref. 12 the system (10) was solved numerically for regular part of the potential (4-6), and irregular terms were calculated as a first order of the perturbation theory. Hereinafter we study the contribution of the singular terms in the framework of the perturbation theory and CIA. We generalize these methods to the system of equations. For this purpose that is convenient to rewrite the system (10) in a matrix form. The Hamiltonian of the system can be separated to regular and irregular parts $H = H_0 + W$, namely

$$\widehat{H_0} = \begin{pmatrix} -\frac{1}{m}\frac{d^2}{dr^2} - \frac{\alpha}{r} + kr + \frac{\beta_V}{3m^2r}; & \sqrt{8}\frac{\beta_V}{m^2r}; \\ \sqrt{8}\frac{\beta_V}{m^2r}; & -\frac{1}{m}\frac{d^2}{dr^2} - \frac{\alpha}{r} + kr + \frac{\beta_V}{3m^2r} + \frac{6}{mr^2} - \frac{2\beta_V}{m^2r} - \frac{3}{m^2}(\frac{3\beta_V - \beta_S}{r}); \end{pmatrix} \tag{11}$$

and

$$\widehat{W} = \begin{pmatrix} \frac{2\pi\alpha}{3m^2}\delta(\overrightarrow{r}); & \sqrt{8}\frac{3\alpha}{m^2r^3}; \\ \sqrt{8}\frac{3\alpha}{m^2r^3}; & \sqrt{8}\frac{3\alpha}{m^2r^3} - \frac{6\alpha}{m^2r^3} - \frac{9\alpha}{2m^2r^3}; \end{pmatrix} \tag{12}$$

Then unperturbated eigenfunction of equation

$$H_0\Psi_n = E_n\Psi_n \tag{13}$$

where

$$\Psi_n = \begin{pmatrix} u_n \\ w_n \end{pmatrix} \tag{14}$$

can be used as a basis for the full wave function of the system, namely,

$$\Phi = \sum a_n\Psi_n \tag{15}$$

Then the matrix elements of the perturbated term

$$W_{mn} = \int \Psi_m^* \widehat{W}\Psi_n d\overrightarrow{r} \tag{16}$$

are given by

$$W_{mn} = \int u_m(-\frac{2\pi\alpha}{3m^2}\delta(\overrightarrow{r}))u_n dr + \int w_m(\sqrt{8}\frac{3\alpha}{m^2r^3})u_n dr + \int u_m(\sqrt{8}\frac{3\alpha}{m^2r^3})w_n dr + \tag{17}$$

$$+\int w_m(-\frac{2\pi\alpha}{3m^2}\delta(\overrightarrow{r}) - \frac{6\alpha}{m^2r^3} - \frac{39\alpha}{2m^2r^3})w_n = W_{SS}^{u_m u_n} + W_{ST}^{w_m u_n} + W_{ST}^{u_m w_n} + W_{SS+LS+ST}^{w_m w_n}$$

and in the framework of the first order of perturbation theory the correction to the energy spectrum is equal to

$$\Delta E_{nn} = W_{SS}^{u_n u_n} + 2W_{ST}^{w_n u_n} + W_{SS+LS+ST}^{w_n w_n} \tag{18}$$

It can be noted that mean value of unperturbed Hamiltonian H_0 (11) consist of the same parts determined by S-wave, SD-interferention and D-wave correspondent

$$E^{(0)} = \langle\psi|H_0|\psi\rangle = E^S + E^{SD} + E^D. \tag{18a}$$

As seen from Eq. (17), the S-wave gives pure contribution only for the spin-spin interaction (the first term), the interferential SD term (the second and third ones) contains the spin-tensor correction, and pure D-wave (the fourth term) includes all the spin-dependent components of the interaction potential. This structure is reflected in Eq. (18) by sub- and superscripts. In the case of the configuration interaction method we used an algorithm, developed in Ref. 3,4. The mean value E of Hamiltonian $H_0 + W$ is an eigenvalue of the following system of linear algebraic equations:

$$\begin{gathered} a_1(E - E_1^0 - W_{11}) - a_2W_{12} - a_3W_{13} - ... - a_nW_{1n} = 0 \\ -a_1W_{21} + a_2(E - E_2^0 - W_{22}) - a_3W_{23} - ... - a_nW_{2n} = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ -a_1W_{n1} - a_2W_{n2} - a_3W_{n3} - ... + a_n(E - E_n^0 - W_{nn}) = 0 \end{gathered} \tag{19}$$

where E_i^0- are eigenvalues of Hamiltonian H_0 (11) and W_{ij} - are correspondent matrix elements (16). Respectively, eigenvectors $(a_1, a_2, ..., a_n)$ gives eigenfunctions Φ_i, $i = 1, 2, ..., n$ of the Hamiltonian $H_0 + W$.

3 Hyperfine and fine splitting

To study the role of tensor forces we calculated for triplet states $L = J \pm 1$ in mixed form (10), and for single S-wave (as most authors do). It was pointed [14,16] that the best agreement with experimental data for potential (1) is obtained when $\beta = 0.18 GeV^2$. For description of $q\overline{q}$-system we were varying parameter β_V (and fixing $\beta_V + \beta_S = 0.18 GeV^2$)) to achieve the agreement with experimental mass splitting of 1^{--} states. Finally we have used the following parameters: $\beta_V = 0.001 GeV^2, \beta_S = 0.179 GeV^2$ for $u\bar{u}$-systems; $\beta_V = 0.04 GeV^2, \beta_S = 0.14 GeV^2$ for charmonium and bottonium, $\alpha_s(b\bar{b}) = 0.24, \alpha_s(c\bar{c}) = 0.38, \alpha_s(u\bar{u}) = 0.54$. The quark masses are: $m_b = 4.7 GeV$ $m_c = 1.4 GeV$, and $m_u = 0.33 GeV$. Tables 2 - 4 list numerical results, namely, mass spectra for single S-wave and mixed state, fraction of tensor part of the potential to energy levels (18a) and D-wave fraction in the total wave function ($P_D = \int |w|^2 dr$). And for comparison we showed experimental values [19] and calculation with screened potential[3,4].

Table 2.Hyperfine splitting for the charmonium

State	S-wave E_{theor}, MeV	SD-waves E_{theor}, MeV	[3] E_{theor}, MeV	[19] E_{exp}, MeV	E^{SD}, %	P_D, %
1^1S_0	2980			2980		
1^3S_1	3153	3097		3097	16	0.05
$1^3S_1 - 1^1S_0$	173	117	110	117		
2^1S_0	3642			3590		
2^3S_1	3759	3734		3685	3	0.8
$2^3S_1 - 2^1S_0$	117	92	67	95		
3^1S_0	4107			-		
3^3S_1	4208	4192		4040	1	1.3
$3^3S_1 - 3^1S_0$	101	85		-		

Table 3.Hyperfine splitting for the bottonium

State	S-wave E_{theor}, MeV	SD-waves E_{theor}, MeV	[3] E_{theor}, MeV	[19] E_{exp}, MeV	E^{SD}, %	P_D, %
1^1S_0	9415			-		
1^3S_1	9462	9460		9460	2.1	0.004
$1^3S_1 - 1^1S_0$	47	45	46	-		
2^1S_0	9883			-		
2^3S_1	9911	9911		10023	0.2	0.04
$2^3S_1 - 2^1S_0$	28	28	26	-		
3^1S_0	10201			-		
3^3S_1	10224	10224		10355	0.1	0.1
$3^3S_1 - 3^1S_0$	23	23		-		

Table 4.Hyperfine splitting for $(u\vec{u})$-systems

State	S-wave E_{theor}, MeV	SD-waves E_{theor}, MeV	[3] E_{theor}, MeV	[19] E_{exp}, MeV	E^{SD}, %	P_D, %
1^1S_0	140			140		
1^3S_1	674	640		770	4	0.02
$1^3S_1 - 1^1S_0$	534	500	923	630		
2^1S_0	1134			1300		
2^3S_1	1564	1543		1450	1	0.04
$2^3S_1 - 2^1S_0$	430	409	411	150		

It can be noted that we obtain good description of mass spectra and hyperfine splitting for heavy quark systems. For light system our quasirelativistic model is less success, here relativistic kinematics play sufficient role [6].

In Tables 5-7 fine splitting data are listed for P-states, where we indicate in brackets result with mixed tensor states ($^3P_2 -^3 F_2$ mixing). Comparison results for $S-D$ and $P-F$ system show that influence of tensor forces decreasing when total moment of the system increases.

Table 5.Fine splitting for $c\bar{c}-$ systems (in MeV)

State	$\triangle M$	Our results	[7]	[20]	[21]	[22]	[19]
$1P$	$M(^3P_2) - M(^3P_1)$	51(49)	576	51	45	28	45.67
	$M(^3P_1) - M(^3P_0)$	72	76	83	91	32	95.51
	$M(^3P_2) - M(^3P_0)$	123(121)	132	134	137	66	141.18

Table 6. Fine splitting for $b\bar{b}-$ systems (in MeV)

State	$\triangle M$	Our results	[7]	[20]	[21]	[21]	[19]
$1P$	$M(^3P_2) - M(^3P_1)$	12(11)	23	31	24	28	19.9 ± 1.1
	$M(^3P_1) - M(^3P_0)$	15	26	41	37	32	32.8 ± 1.5
	$M(^3P_2) - M(^3P_0)$	27(26)	49	72	61	66	52.7 ± 1.5
$2P$	$M(^3P_2) - M(^3P_1)$	10 (10)	16	24	17	20	13.3 ± 0.9
	$M(^3P_1) - M(^3P_0)$	15	20	32	26	24	23.1 ± 1.1
	$M(^3P_2) - M(^3P_0)$	25(25)	36	56	44	44	36.4 ± 1.0

Table 7. Hyperfine splitting in $P-$waves (in MeV)

State	$\triangle M$	Our results	[7]	[19]
$c\bar{c}(1P)$	$M(^3P_1) - M(^1P_1)$	-10	13	-15.63 ± 0.36
$b\bar{b}(1P)$	$M(^3P_1) - M(^1P_1)$	-3.4	4.3	$---$
$b\bar{b}(2P)$	$M(^3P_1) - M(^1P_1)$	-3.5	3	$---$

4 Vector mesons in CI approach

We also analyzed influence of perturbation part of the system Hamiltonian because most authors study spin-spin interaction in the frame of perturbation theory of the first order. To estimate high order theory we use the configuration interaction approach (CIA), the one configuration case of which coincides with the first order perturbation theory.

Below the numerical results for the mass spectrum of $c\bar{c}$ and $u\bar{u}$ systems are given. The results of the mass spectrum calculations listed in Table 8 (nonperturbated eigenvalues of energy, correction according to the perturbation theory, energies according to CIA with two and three configurations). It is seen from the table that the series of Eq. (10) converges rapidly for heavy systems and already the perturbation theory gives more than 90% contribution of the singular terms, but for light mesons the extension beyond the perturbation theory is essential.

Table 8. Vector meson mass spectrum

States	$\mathbf{M}_{TH}, MeV$ non-perturb.	$\mathbf{M}_{TH}, MeV$ 1 config.	$\mathbf{M}_{TH}, MeV$ 2 config.	$\mathbf{M}_{TH}, MeV$ 3 config.	$\mathbf{M}_{EXP}, MeV$
$J/\psi(1S)$	3043.45	3098.58	3097.11	3096.87	3096.87
$\psi(2S)$	3649.04	3674.02	3675.50	3674.87	3685.96
$\psi(4040)$	4098.04	4113.11	$---$	4113.97	4040
$\rho(770)$	685.5	778	771	768.5	768.5
$\rho(1450)$	1574.5	1643	1650	1643	1465
$\rho(1700)$	2270.5	2332	$---$	2341	1700

Table 9.Perturbation theory for a J/ψ meson

	$\mathbf{W_{SS}^{u_n u_n}}, \mathbf{MeV}$	$\mathbf{2W_{ST}^{w_n u_n}}, \mathbf{MeV}$	$\mathbf{W_{SS+ST+LS}^{w_n w_n}}, \mathbf{MeV}$
$\mathbf{\Delta E_{11}}, \mathbf{MeV}$	6.04	49.6	0.00050
$\mathbf{\Delta E_{22}}, \mathbf{MeV}$	4.18	21.16	0.00036
$\mathbf{\Delta E_{33}}, \mathbf{MeV}$	3.65	11.95	0.00052

Table 10.Perturbation theory for a ρ meson

	$\mathbf{W_{SS}^{u_n u_n}}, \mathbf{MeV}$	$\mathbf{2W_{ST}^{w_n u_n}}, \mathbf{MeV}$	$\mathbf{W_{SS+ST+LS}^{w_n w_n}}, \mathbf{MeV}$
$\mathbf{\Delta E_{11}}, \mathbf{MeV}$	60.98	31.67	-0.19
$\mathbf{\Delta E_{22}}, \mathbf{MeV}$	50.19	19.02	-0.28
$\mathbf{\Delta E_{33}}, \mathbf{MeV}$	46.64	14.69	-0.075

The absolute values of corrections due to certain singular terms of the potential in the framework of the perturbation theory are rather interesting. The correction values for the spin-spin, spin-tensor and spin-orbit components of the singular part of the interaction for charmonium and ρ meson are given in Tables 9 and 10, respectively. It is seen that for heavy systems the spin-tensor part of the correction value by order of magnitude exceeds the spin-spin part and is totally determined by the presence of the D wave admixture. For light systems the spin-spin and spin-tensor correction values are of the same order. Hence, when the spin effects are considered, one should take into account the orbital structure of the meson states, especially for the systems of heavy quarks. However, we note that the D-wave admixture for the light and heavy systems is less than 1 % and about 4% , respectively [12]. In comparison with Ref. 10 , we note that for the $\Psi(2S+D)$ state the D-wave admixture in our case is $P_D = 0.008$ [16], which corresponds to the mixing angle of $\varphi = 5^o$. In Ref. 7 the mixing angle of pure triplet states $2S$ and $1D$ $\varphi = 12^o$ is quoted.

5 Leptonic decay of heavy quarkonia and wave function in the origin

For the leptonic decay widths of two-quark system we shall consider decay of vector states into e^+e^- pairs. The leptonic decay width of system $M_{q\bar{q}} \to e^+e^-$ is calculated from the Van Royen-Weisskopf formula [23]

$$\tilde{\Gamma}(^3S_1 \to e^+e^-) = \frac{4\alpha_{em}^2 Q^2}{M_{q\bar{q}}^2}|R(0)|^2, \tag{20}$$

where $M_{q\bar{q}}$ is mass of vector meson, Q is quark charge, α_{em} is the fine structure constant and $R(0)$ is the radial wave function in the origin. In our case radial wave function in the origin is determined by S-wave component of the system wave function, namely, by $\frac{u(r)}{r}$ (9) because the D-wave component vanishes in the origin.

The formula (20) is based on the notion that constituent quark-antiquark pair annihilates into a single virtual photon, which subsequently gives rise to a leptonic pair. The relativistic and radiative QCD corrections [24] modifies eq.(20)

$$\Gamma(^3S_1 \rightarrow e^+e^-) = \tilde{\Gamma}(1 - \frac{16\alpha_s(m_q^2)}{3\pi}). \tag{21}$$

As Eichten and Quigg have pointed out [25] the QCD correction reduces the magnitude of Γ significantly, however the value of reduction is somewhat uncertain . For vector mesons containing light quarks this formula leads to paradoxes [26]. In paper [27] Motyka and Zalewski extrapolated eq. (21) by rational and exponential function and obtain averaged formula.

$$\Gamma_{V\rightarrow e^+e^-} = F(q)\frac{32\alpha_s}{9M_V^2}|R(0)|^2, \tag{22}$$

with $F(c) = 4.73.10^{-5}$ for charmonium and $F(b) = 2.33.10^{-5}$ for bottonium. We have calculated decay widths using the formula of Van Royen-Weisskopf (20) and formula (22). Table 11 lists this results.

Table 11. The leptonic decay widths of heavy mesons

State	S-waves $\Gamma_{theor.}$,keV	SD-wave $\Gamma_{theor.}$,keV	[27] $\Gamma_{theor.}$,keV	[5] $\Gamma_{theor.}$,keV	[27] $\Gamma_{theor.}$,keV	[19] $\Gamma_{exp.}$,keV
$J/\psi 1S$	8.2 (5.63)	7.8 (5.41)	4.5	4.24	8.0	5.26 ± 0.37
$\psi' 2S$	4.0 (2.79)	3.7 (2.59)	1.9	1.81	3.7	2.12 ± 0.18
$\psi'' 3S$	2.9 (2.01)	2.6(1.82)	- - -	1.22	- - -	0.75 ± 0.15
$\Upsilon 1S$	1.2 (1.01)	1.14(0.96)	1.36	0.85	1.7	1.32 ± 0.04
$\Upsilon' 2S$	0.63 (0.53)	0.58(0.49)	0.59	0.38	0.8	0.52 ± 0.03
$\Upsilon'' 3S$	0.49 (0.42)	0.44(0.37)	0.40	0.27	0.6	0.48 ± 0.08

The calculations of widths were performed with tensor forces and without ones. Value of the widths which were calculated by formula (22) are given in parentheses.

Table 13. Wave functions in the origin $|R(0)|^2$. (In GeV^3)

State	$\lvert R(0)\rvert^2$, non-perturb.	$\lvert R(0)\rvert^2$, 1 config.	$\lvert R(0)\rvert^2$, 2 config.	$\lvert R(0)\rvert^2$, 3 config.	$\lvert R(0)\rvert^2$, [5]	$\lvert R(0)\rvert^2$, [25]	$\lvert R(0)\rvert^2$, [18]
$J/\psi(1S)$	0.77	0.69	0.83	.85	1.05	1.45	0.81
$\psi(2S)$	0.53	0.56	0.47	0.50	0.63	0.93	0.53
$\psi(4040)$	0.47	0.53	$---$	0.41	0.52	0.79	0.46
$\rho(770)$	0.11	0.086	0.126	0.136	$---$	$---$	$---$
$\rho(1450)$	0.091	0.081	0.073	0.089	$---$	$---$	$---$
$\rho(1700)$	0.084	0.12	$---$	0.061	$---$	$---$	$---$

The values of the squared radial wave function in the origin for the non-perturbed and the perturbed case along with the CIA calculations with two and three configuration sets are listed in Table 13. It is seen that, contrary to the energy spectra, the consideration of the decay widths beyond the framework of the perturbation theory results in the correction value of 20-30% . Our results are very close to those of Ref. 17 where the same quark systems were considered on the base of Schroedinger equation with the generalized Breit-Fermi potential. The difference from our work is in the choice of the Lorentz structure of the quark-quark interaction and neglecting the D-wave admixture. Also squared mean value radius obtained from our wave function coincide well with other calculations (Table 12).

Table 12. Squared mean value radius

nL	[22] $\langle r_c^2\rangle^{1/2}$, fm	Our results $\langle r_c^2\rangle^{1/2}$,fm	[29] $\langle r_b^2\rangle^{1/2}$, fm	Our results $\langle r_b^2\rangle^{1/2}$, fm
1 SD	0.43	0.433	0.24	0.256
2 SD	0.85	0.847	0.51	0.552
3 SD	1.18	1.182	0.73	0.768

It must be noted that squared mean value radius 0.7 fm meet the requirements of quark-antiquark pair creation ($u\bar{u}$) (string break). It is necessary to modify the potential model taking into account the opening of a new channel for those conditions where the given value is overlapped.

6 Conclusions

In the present work we have studied the influence of the quark-antiquark interaction potential structure to the meson properties. We pay the main attention to the role of the tensor forces. It was considered mass spectra and leptonic decay width.

The regular part of the potential was taken into account by numerical solutions of Schroedinger equations for singlet and unmixed triplet states but a coupled differential equation for mixed triplet states ($S-D$- for vector mesons and $P-F$- for tensor mesons). The irregular part of potential was taken into account in the frame of the configuration interaction approach.

The analysis of our results presented in Table 2 - 4 shows that differences between the theoretical calculations of mass spectrum for heavy quarkonia and the experimental results are 1 - 4 %. Therefore we expect relativistic correction in the range 1 - 16 % for charmonium, and up to 1 % for bottonium spectrum. We have also been able to describe hyperfine splitting of $c\bar{c}$- and $b\bar{b}$-quarkonium. For describing mass spectrum of light mesons it is necessary to use relativistic potential model. We have calculated hyperfine splitting $u\bar{u}$ system, too.
It shown that, admixture of D-wave is small (less then 1%) but its influence to the mass spectra reach 16%. The leptonic decay widths suggest that for charmonium theoretical widths, which were calculated by Van Royen-Weisskopf formula are systematically higher than experimental data. But for bottonium we have obtained lower values. For J/ψ-meson better decay widths are obtained by means of formula (21), which takes into account QCD correction. Here the influence of D-waves ranges from 4 % for ground state up to 25 % for the second excited state, but for values calculated by Van Royen-Weisskopf formula it is from 8 % up to 50 %. For Υ-meson it is opposite. More exact is the calculation done by (21), but QCD correction reduced the results more significantly. Besides, D-waves contribute less than for charmonium: from 4 % for ground state up to 11 % for second excited state.

The results obtained show that contribution of the D-waves is impossible to neglect for considered leptonic decay width of quarkonia.

It is shown that, in spite of a small admixture of the D-wave this component of the wave function plays an essential role at the account of the irregular part of the interaction potential, and namely the irregular terms are considered by most authors while the spin effects being studied. The presence of the D -wave essentially enhances the contribution of the spin-tensor component into the mass spectrum. As concerning the technique of the account of the irregular part of the interaction it should be noted that in case the mass spectrum of the systems being considered, one can restrict themselves by the first order correction of the perturbation theory, but at the analysis of the decay widths the CIA results essentially improve the first order perturbation theory.

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References

[1] C. Quigg, J. L. Rosner, Phys. Rep. **56**, 167 (1982)

[2] G. Altarelli, N. Cabibo, G. Corbo, L. Maiani and G. Martinelli, Nucl. Phys. **B 208**, 365 (1982)

[3] V. Lengyel,Yu. Fekete, I. Haysak, A. Shpenik, Eur.Phys.J. **C 21** 355 (2001)

[4] V. Lengyel, V. Rubish, A. Shpenik, Ukr. Phys. J. **47**, 508 (2002)

[5] V. Lengyel, V. Makkay, S. Chalupka, V. Salak, Ukr. Phys. J. **42**, 773 (1997)

[6] V. Lengyel, V. Rubish, Yu. Fekete, S. Chalupka, M. Salak, J. Phys. Stud. **2**, 38 (1998)

[7] S. Chalupka, V. Lengyel, P.Petreczky, F. Paccanoni, M. Salak, Nuovo Chimento **A 107**, 1557 (1994)

[8] Ajay K. Ray, R. H. Parmar, P. C. Vinodkumar, J. Phys. G: Nucl. Part. Phys. **28**, 2275 (2002)

[9] Moxhay, J.L. Rosner, Phys. Rev **D 28**, 1132(1983)

[10] J. L. Rosner, Phys. Rev D, Vol. 64, 094002

[11] R. McClary, N. Byers, Phys. Rev. **D 28**, 1692 (1983)

[12] I. I. Haysak, V.S. Morokhovych, J. Phys. Stud. V. 6, 55, (2002); I.Haysak, V. Morokhovych, S. Chalupka, M. Salak, hep-ph/0201038

[13] D. Ebert, R. N. Faustov, V. O. Galkin, Eur. Phys. J. **C7**, 539 (1999)

[14] D. Ebert, V. O. Galkin and R. Faustov, Phys. Rev. **D 57**, 5663 (1998)

[15] W. Buchmüller, Phys. Rev. Lett. **B 112**, 479(1982)

[16] I. Haysak, V. Lengyel, V. Morokhovych, Proceeding of the International Conference. Small Triangle Meeting Kosice, Slovak Republic, 8-9 september 1999, pp. 113 - 119

[17] W. Rarita, J. Schwinger, Phys.Rev.**59**,436 (1941)

[18] Buchmuller W., Tye S.-H., Phys. Rev. **D24**, 132 (1981)

[19] Part. Data Group, The European Physical Journal, Vol 15, 1 (2000)

[20] W. Buchmuller, Phys. Rev. Lett. **B 112**, 479 (1982)

[21] M. Hirano, Prog. Theor. Phys. **83**, 575 (1990)

[22] H. W. Crater and P. Van Alstin, Phys. Rev. **D 37**, 1982 (1987)

[23] R. Van Royen and V. Weisskopf, Nuovo Cim. **A50** 2566 (1967)

[24] R. Barbieri et al, Nucl. Phys. **B 154**, 535 (1979)

[25] E. Eichten, C. Quigg , Phys. Rev. **D 52**, 1726 (1995)

[26] S. Narison and K. Zalewski, Phys. Lett. **B 320**, 369 (1994)

[27] L. Motyka, K.Zalevski, Eur. Phys. J. **C 4**, 107 (1998)

[28] E. Eichten and C. Quigg, Phys. Rev. **D 49**, 5845 (1994)

[29] G. S. Bali, A. Watcher, K. Schilling, Phys. Rev **D 56**, 2566 (1997)